Write your name here

Surname	Other names

Pearson Edexcel
International Advanced Level

Centre Number | Candidate Number

Core Mathematics C12

Advanced Subsidiary

Monday 10 October 2016 – Morning
Time: 2 hours 30 minutes

Paper Reference
WMA01/01

You must have:
Mathematical Formulae and Statistical Tables (Blue)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 125.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

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1.

$$f(x) = 3x^2 + x - \frac{4}{\sqrt{x}} + 6x^{-3}, \quad x > 0$$

Find $\int f(x)\,dx$, simplifying each term.

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Question 1 continued

(Total 5 marks)

Q1

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2. Find, giving your answer to 3 significant figures where appropriate, the value of x for which

(a) $7^{2x} = 14$

(3)

(b) $\log_5(3x + 1) = -2$

(2)

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Question 2 continued

(Total 5 marks)

Q2

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3. Answer this question without the use of a calculator and show your method clearly.

(i) Show that

$$\sqrt{45} - \frac{20}{\sqrt{5}} + \sqrt{6}\sqrt{30} = 5\sqrt{5}$$

(2)

(ii) Show that

$$\frac{17\sqrt{2}}{\sqrt{2} + 6} = 3\sqrt{2} - 1$$

(3)

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Question 3 continued

Q3

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4. $$f(x) = 6x^3 - 7x^2 - 43x + 30$$

(a) Find the remainder when $f(x)$ is divided by

(i) $2x + 1$

(ii) $x - 3$

(4)

(b) Hence factorise $f(x)$ completely.

(4)

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Question 4 continued

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Question 4 continued

Q4

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5. (a) Find the first 4 terms, in ascending powers of x, of the binomial expansion of

$$\left(3 - \frac{ax}{2}\right)^5$$

where a is a positive constant. Give each term in its simplest form.

(4)

Given that, in the expansion, the coefficient of x is equal to the coefficient of x^3,

(b) find the exact value of a in its simplest form.

(3)

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Question 5 continued

Q5

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6. A sequence is defined by

$$u_1 = 36$$
$$u_{n+1} = \frac{2}{3}u_n, \qquad n \geqslant 1$$

(a) Find the exact simplified values of u_2, u_3 and u_4 **(2)**

(b) Write down the common ratio of the sequence. **(1)**

(c) Find, giving your answer to 4 significant figures, the value of u_{11} **(2)**

(d) Find the exact value of $\sum_{i=1}^{6} u_i$ **(2)**

(e) Find the value of $\sum_{i=1}^{\infty} u_i$ **(2)**

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Question 6 continued

Q6

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7. (a) Sketch the graph of $y = 3^{x-2}$, $x \in \mathbb{R}$
Give the exact values for the coordinates of the point where your graph crosses the y-axis.

(2)

The table below gives corresponding values of x and y, for $y = 3^{x-2}$
The values of y are rounded to 3 decimal places where necessary.

x	0.5	1	1.5	2	2.5	3
y	0.192	0.333	0.577	1	1.732	3

(b) Use the trapezium rule with all the values of y from the table to find an approximate value for

$$\int_{0.5}^{3} 3^{x-2}\,\mathrm{d}x$$

Give your answer to 2 decimal places.

(4)

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Question 7 continued

Q7

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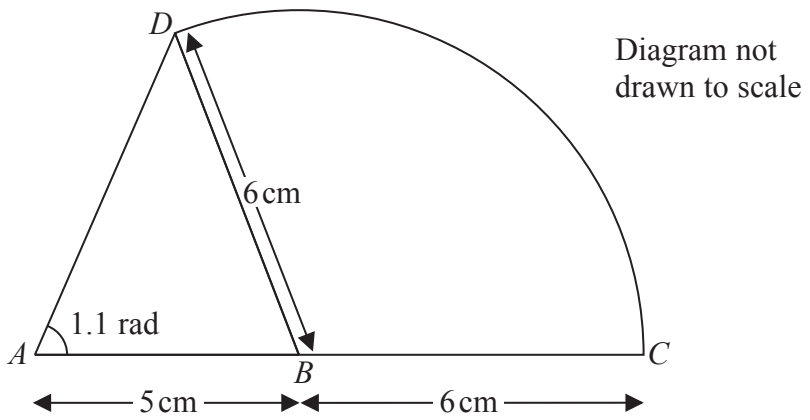

Figure 1

The compound shape *ABCDA*, shown in Figure 1, consists of a triangle *ABD* joined along its edge *BD* to a sector *DBC* of a circle with centre *B* and radius 6 cm.
The points *A*, *B* and *C* lie on a straight line with $AB = 5$ cm and $BC = 6$ cm.
Angle $DAB = 1.1$ radians.

(a) Show that angle $ABD = 1.20$ radians to 3 significant figures.

(4)

(b) Find the area of the compound shape, giving your answer to 3 significant figures.

(4)

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Question 8 continued

Q8

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9. In a large theatre there are 20 rows of seats.

The number of seats in the first row is a, where a is a constant.

In the second row the number of seats is $(a + d)$, where d is a constant. In the third row the number of seats is $(a + 2d)$, and on each subsequent row there are d more seats than on the previous row. The number of seats in each row forms an arithmetic sequence.

The **total** number of seats in the first 10 rows is 395

(a) Use this information to show that $10a + 45d = 395$

(1)

The **total** number of seats in the first 18 rows is 927

(b) Use this information to write down a second simplified equation relating a and d.

(2)

(c) Solve these equations to find the value of a and the value of d.

(3)

(d) Find the number of seats in the 20th row of the theatre.

(2)

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Question 9 continued

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10. (a) Given that

$$8\tan x = -3\cos x$$

show that

$$3\sin^2 x - 8\sin x - 3 = 0$$

(3)

(b) Hence solve, for $0 \leqslant \theta < 360°$,

$$8\tan 2\theta = -3\cos 2\theta$$

giving your answers to one decimal place.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

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Question 10 continued

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11. The equation $5x^2 + 6 = k(13x^2 - 12x)$, where k is a constant, has two distinct real roots.

(a) Show that k satisfies the inequality

$$6k^2 + 13k - 5 > 0$$

(4)

(b) Find the set of possible values for k.

(4)

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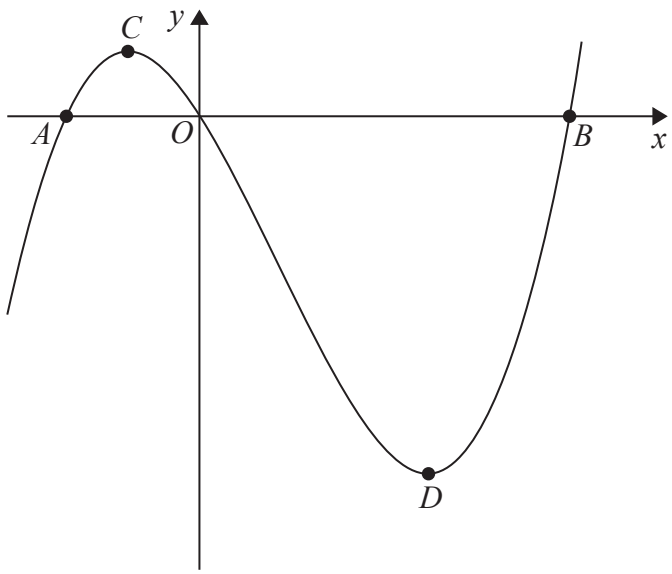

Diagram not drawn to scale

Figure 2

Figure 2 shows a sketch of the curve with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = \frac{x^3 - 9x^2 - 81x}{27}$$

The curve crosses the x-axis at the point A, the point B and the origin O.
The curve has a maximum turning point at C and a minimum turning point at D.

(a) Use algebra to find exact values for the x coordinates of the points A and B.

(4)

(b) Use calculus to find the coordinates of the points C and D.

(6)

The graph of $y = \mathrm{f}(x + a)$, where a is a constant, has its minimum turning point on the y-axis.

(c) Write down the value of a.

(1)

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Question 12 continued

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13. The circle C has centre $A(1, -3)$ and passes through the point $P(8, -2)$.

(a) Find an equation for the circle C.

(4)

The line l_1 is the tangent to C at the point P.

(b) Find an equation for l_1, giving your answer in the form $y = mx + c$

(4)

The line l_2, with equation $y = x + 6$, is the tangent to C at the point Q.

(c) Find the coordinates of the point Q.

(5)

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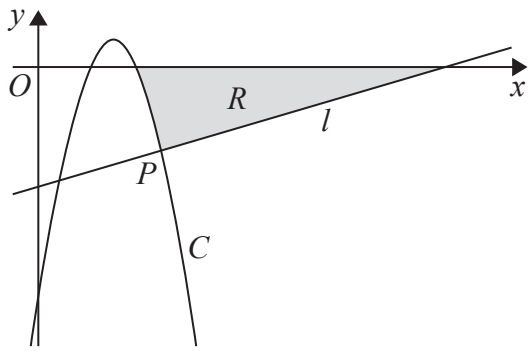

Figure 3

Figure 3 shows a sketch of the curve C with equation $y = -x^2 + 6x - 8$. The normal to C at the point $P(5, -3)$ is the line l, which is also shown in Figure 3.

(a) Find an equation for l, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(5)

The finite region R, shown shaded in Figure 3, is bounded below by the line l and the curve C, and is bounded above by the x-axis.

(b) Find the exact value of the area of R.

(6)

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

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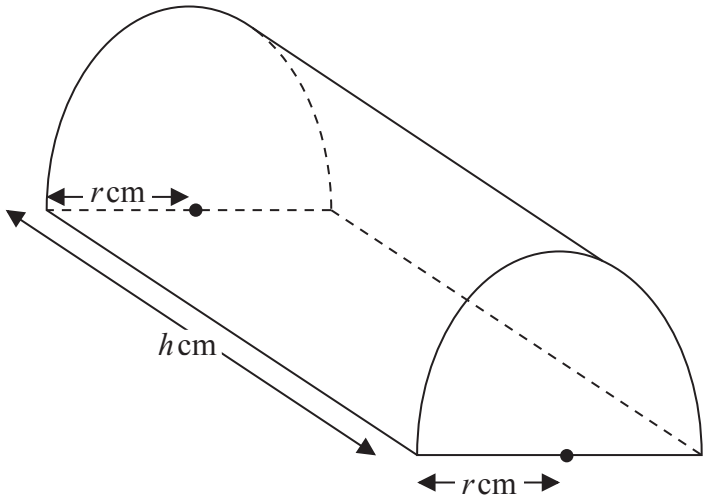

Figure 4

Figure 4 shows a solid wooden block. The block is a right prism with length h cm. The cross-section of the block is a semi-circle with radius r cm.

The total surface area of the block, including the curved surface, the two semi-circular ends and the rectangular base, is 200 cm^2

(a) Show that the volume V cm^3 of the block is given by

$$V = \frac{\pi r(200 - \pi r^2)}{4 + 2\pi}$$

(5)

(b) Use calculus to find the maximum value of V. Give your answer to the nearest cm^3.

(6)

(c) Justify, by further differentiation, that the value of V that you have found is a maximum.

(2)

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